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(Non-)Abelian-gauged supergravities in nine dimensions

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Abstract

We construct five massive deformations of the unique nine-dimensional $N = 2$ supergravity, each with two parameters. All these deformations have a higher-dimensional origin via Scherk–Schwarz reduction and correspond to gauged supergravities. The gauge groups we encounter are $SO(2)$, $SO(1, 1)^{+}$, $\mathbb{R}$, $\mathbb{R}^{+}$ and the two-dimensional non-Abelian Lie group $A(1)$, which consists of scalings and translations in one dimension.

We make a systematic search for (non-supersymmetric) de Sitter space solutions. Furthermore, we discuss which of the $D = 9$ gauged supergravities can be considered as candidate low-energy limits of compactified superstring theory.

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1. Introduction

The procedure of gauging a global symmetry includes the replacement of the ordinary derivative by a covariant derivative:

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + gA_\mu. \tag{1}$$

Here A_μ is the gauge field and g is the gauge coupling constant which acts as a deformation parameter of the ungauged theory. In the case of Einstein gravity with scalars one can consider as an independent deformation the addition of a scalar potential $V(\varphi)$:

$$R + (\partial\varphi)^2 \rightarrow R + (\partial\varphi)^2 + m^2 V(\varphi). \tag{2}$$

In the supersymmetric case, i.e., the case of *gauged* supergravity, the two deformations are not independent. Supersymmetry relates the two deformation parameters:

$$g = m. \tag{3}$$

Due to the scalar potential the Minkowski spacetime is no longer a maximally supersymmetric vacuum solution of the gauged supergravity. Instead we will search for

Table 1. The scaling weights of the nine-dimensional supergravity fields.

	$e_\mu{}^a$	e^ϕ	e^φ	χ	A_μ	$A_\mu^{(1)}$	$A_\mu^{(2)}$	$B_{\mu\nu}^{(1)}$	$B_{\mu\nu}^{(2)}$	$C_{\mu\nu\rho}$	ψ_μ	λ	$\tilde{\lambda}$
α	$\frac{9}{7}$	0	$\frac{6}{\sqrt{7}}$	0	3	0	0	3	3	3	$\frac{9}{14}$	$-\frac{9}{14}$	$-\frac{9}{14}$
β	0	$\frac{3}{4}$	$\frac{\sqrt{7}}{4}$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0
γ	0	-2	0	2	0	1	-1	1	-1	0	0	0	0
δ	$\frac{8}{7}$	0	$-\frac{4}{\sqrt{7}}$	0	0	2	2	2	2	4	$\frac{4}{7}$	$-\frac{4}{7}$	$-\frac{4}{7}$

other vacuum solutions, such as, e.g., non-supersymmetric de Sitter space solutions. We consider $D = 9$ dimensions because on the one hand this case shares some of the complexities of the lower-dimensional cases, and on the other hand, the scalar potential for this case is simple enough to study possible vacuum solutions in full detail. The supergravity theory we considered in [1] was obtained by a generalized Scherk–Schwarz (SS) reduction of $D = 10$ IIB supergravity. This is not the most general possibility in $D = 9$. In this paper we will present a systematic search for massive deformations of the unique $D = 9$, $N = 2$ supergravity theory. All deformations we find correspond to *gauged* supergravities. The hope is that the $D = 9$ case will teach us something about the more complicated situation in $D < 9$ dimensions. The results presented in this paper are taken from [2].

2. Massive deformations of $D = 9$, $N = 2$ supergravity

The field content of the unique $D = 9$, $N = 2$ massless supergravity theory is given by ($i = 1, 2$)

$$e_\mu{}^a, \phi, \varphi, \chi, A_\mu, A_\mu^{(i)}, B_{\mu\nu}^{(i)}, C_{\mu\nu\rho}, \psi_\mu, \lambda, \tilde{\lambda}. \tag{4}$$

The massless nine-dimensional (9D) theory has four global scaling symmetries, with parameters α, β, γ and δ, respectively. The scaling weights of all these symmetries are given in table 1.

It turns out that only three out of the four scaling symmetries given in table 1 are linearly independent. There is a relation

$$\tfrac{4}{9}\alpha - \tfrac{8}{3}\beta = \gamma + \tfrac{1}{2}\delta. \tag{5}$$

We now turn to massive deformations of the 9D theory. To obtain these deformations we will apply a SS reduction which can be best illustrated by an example. Consider a single scalar field coupled to gravity

$$\hat{\mathcal{L}} = -\tfrac{1}{2}\sqrt{-\hat{g}}(\partial\hat{\phi})^2, \tag{6}$$

which is invariant under the $\mathbb{R}$-symmetry $\hat{\phi} \to \hat{\phi} + c$. In the SS procedure, one gives the field a dependence on the compactification coordinate z which is governed by a global symmetry, in this case the $\mathbb{R}$-symmetry:

$$\hat{\phi}(x, z) = \phi(x) + m_\phi z. \tag{7}$$

Using the standard reduction rules[1], the Lagrangian reduces to

$$\mathcal{L} = -\tfrac{1}{2}\sqrt{-g}\big((D\phi)^2 + m_\phi^2\big), \tag{8}$$

where $D_\mu\phi = \partial_\mu\phi - m_\phi A_\mu$ with A_μ being the Kaluza–Klein vector. Thus, we see that the SS reduction procedure leads to a massive deformation which is a gauge theory with the Kaluza–Klein vector A_μ playing the role of the gauge field.

[1] For simplicity, we take the Kaluza–Klein scalar φ equal to 1.

One cannot always perform the SS reduction on the Lagrangian. Sometimes one can apply the SS procedure to the equations of motion only. To explain this point, we consider a slight generalization of the previous example where we also include the Einstein term. The Lagrangian reads

$$\hat{\mathcal{L}} = \sqrt{-\hat{g}}\left[\hat{R} - \tfrac{1}{2}(\partial\hat{\phi})^2\right]. \tag{9}$$

This system has two global symmetries:

$$\hat{g}_{\hat{\mu}\hat{\nu}} \to \mathrm{e}^{2m_g}\hat{g}_{\hat{\mu}\hat{\nu}}, \qquad \hat{\phi} \to \hat{\phi} + m_\phi. \tag{10}$$

The shift of the dilaton is a symmetry of the Lagrangian. The scale transformation of the metric is a symmetry of the field equations only; it scales the Lagrangian.

Using only the metric scaling, i.e., $m_\phi = 0$, we make the following ansatz for Scherk–Schwarz reduction over z to nine dimensions[2]:

$$\hat{e}_{\hat{\mu}}{}^{\hat{a}} = \mathrm{e}^{m_g z}\begin{pmatrix} \mathrm{e}^{\sqrt{7}\varphi/28}e_\mu^a & 0 \\ 0 & \mathrm{e}^{-\sqrt{7}\varphi/4} \end{pmatrix}, \qquad \hat{\phi} = \phi. \tag{11}$$

Using this ansatz the 10D field equations yield the following 9D equations:

$$\begin{aligned}
[\hat{g}^{\mu\nu}]: \quad & R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} - \tfrac{1}{2}\partial_\mu\phi\partial_\nu\phi + \tfrac{1}{4}(\partial\phi)^2 g_{\mu\nu} - \tfrac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \tfrac{1}{4}(\partial\varphi)^2 g_{\mu\nu} \\
& \quad + 28\,\mathrm{e}^{4\varphi/\sqrt{7}}m_g{}^2 g_{\mu\nu} = 0, \\
[\hat{\phi}]: \quad & \phi = 0, \\
[\hat{g}^{zz}]: \quad & \varphi = 0.
\end{aligned} \tag{12}$$

If one performs the SS reduction on the 10D Lagrangian, instead of on the field equations, the result reads $\hat{\mathcal{L}} = \mathrm{e}^{8m_g z}\mathcal{L}$. It turns out that the 9D Lagrangian $\mathcal{L}$ only gives the correct equations of motion (12) if $m_g = 0$. Thus the application of the SS reduction to the Lagrangian does not give the correct answer if the Lagrangian scales. For more details of this example, see appendix B of [2].

Combining the above two examples, we conclude that the Scherk–Schwarz reduction procedure leads to massive deformations corresponding to gauged supergravity theories where the Kaluza–Klein vector plays the role of the gauge field. Furthermore performing the reduction on the Lagrangian is only legitimate when the exploited symmetry leaves the Lagrangian *invariant* rather than covariant. For symmetries that scale the Lagrangian one has to reduce the field equations.

Applying the above outlined SS-dimensional reduction we obtain a number of massive deformations in nine dimensions, as illustrated in figure 1. By employing the different global symmetries of 11D, IIA and IIB supergravity, we obtain seven deformations of the unique $D = 9$ supergravity. For related work, see [3–8].

3. Combining massive deformations

We next try to combine the different massive deformations we found above. The requirement that the fermionic field equations transform under supersymmetry to a complete set of bosonic field equations restricts us to five cases, each containing two non-zero mass parameters:

- *Case 1 with* $\{m_{\mathrm{IIA}}, m_4\}$. This combination can also be obtained by the Scherk–Schwarz reduction of IIA employing a linear combination of the symmetries $\hat{\alpha}$ and $\hat{\beta}$, guaranteeing its consistency. It is a gauging of both this symmetry and (for $m_4 \neq 0$) the parabolic subgroup of $SL(2,\mathbb{R})$ in 9D, giving the non-Abelian-gauge group $A(1)$.

[2] In this example we keep the Kaluza–Klein scalar but set the Kaluza–Klein vector A_μ equal to zero for simplicity.

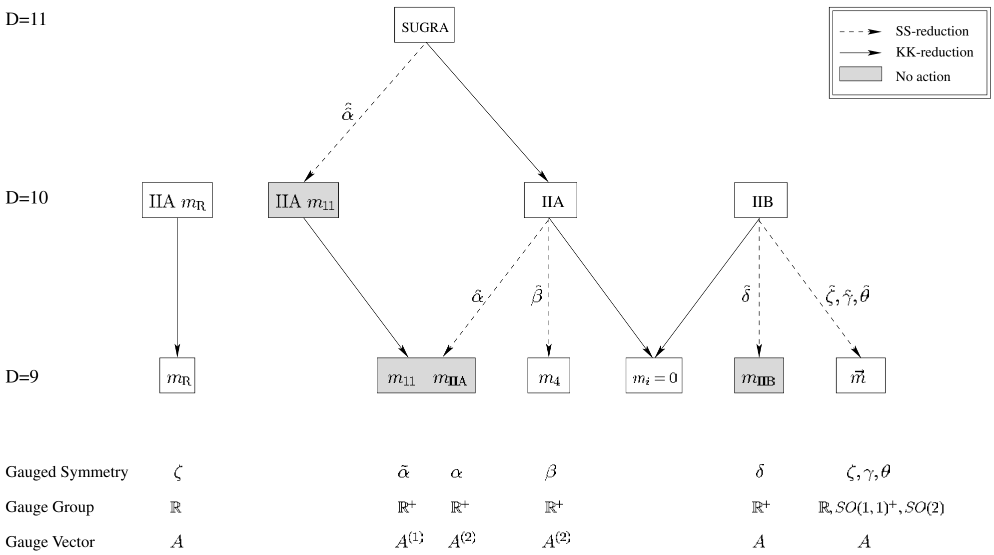

Figure 1. Overview of all reductions discussed in this paper. These cases can all be interpreted as gauged supergravities, with gauged symmetry and corresponding gauge field as given in the figure. Mass parameters in the same box, such as m_{11}, m_{IIA} or m_1, m_2, m_3, form a multiplet under $SL(2, \mathbb{R})$.

- *Cases 2–4 with* $\{\vec{m}, m_{\mathrm{IIB}}\}$. This combination contains three different, inequivalent cases depending on $\vec{m}^2$:

  Case 2 with $\{\vec{m}, m_{\mathrm{IIB}}\}$ and $\vec{m}^2 = 0$.
  Case 3 with $\{\vec{m}, m_{\mathrm{IIB}}\}$ and $\vec{m}^2 > 0$.
  Case 4 with $\{\vec{m}, m_{\mathrm{IIB}}\}$ and $\vec{m}^2 < 0$.

  All these combinations can be obtained by the Scherk–Schwarz reduction of IIB employing a linear combination of the symmetries $\hat{\delta}$ and (one of the subgroups of) $SL(2, \mathbb{R})$, guaranteeing its consistency. All cases (assuming that $m_{\mathrm{IIB}} \neq 0$) correspond to the gauging of an Abelian non-compact symmetry in 9D. In the special case $m_{\mathrm{IIB}} = 0$, cases 2, 3 and 4 correspond to the gauging of $\mathbb{R}$, $SO(1, 1)$ and $SO(2)$ subgroups of $SL(2, \mathbb{R})$, respectively.
- *Case 5 with* $\left\{m_4 = -\frac{12}{5} m_{\mathrm{IIA}}, m_2 = m_3\right\}$. This case can be understood as the generalized dimensional reduction of Romans' massive IIA theory, employing the $\mathbb{R}^+$ symmetry that is not broken by the m_{R} deformations: $\hat{\beta} - \frac{5}{12}\hat{\alpha}$. It gauges both this linear combination of $\mathbb{R}^+$ and the parabolic subgroup of $SL(2, \mathbb{R})$ in 9D, giving a non-Abelian-gauge group provided $m_4 \neq 0$.

All five cases are gauged theories and have a higher-dimensional origin. Both case 1 and case 5 have a non-Abelian-gauge group provided $m_4 \neq 0$.

4. Solutions

We have constructed a variety of gauged supergravities with 32 supersymmetries. They all have in common that there is a scalar potential. Our next goal is to make a systematic search for maximally symmetric solutions with constant scalars, i.e., de Sitter (dS), Minkowski (Mink) or anti-de Sitter (AdS) solutions. For a discussion of the half-supersymmetric domain wall solutions, see [1, 2].

The truncation to all three scalars constant is a consistent truncation in two cases which both have two mass parameters. In this truncation one is left with the metric only satisfying the Einstein equation with a cosmological term

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu}, \tag{13}$$

with Λ quadratic in the two mass parameters. Depending on the sign of this term one thus has anti-de Sitter, Minkowski or de Sitter geometry.

We find that solutions with constant scalars are possible in the following massive supergravities:

- *$D = 10$* with $\{m_{11}\}$ has $\Lambda = +36 {m_{11}}^2 \, \mathrm{e}^{-3\hat{\phi}/2}$, which gives rise to de Sitter$_{10}$ [9], breaking all supersymmetry. The $D = 11$ origin of this solution is Mink$_{11}$ written in a basis where the x-dependence is of the required form [9]:

$$\mathrm{Mink}_{11}: \quad \mathrm{d}s^2 = \mathrm{e}^{2m_{11}x}\left(-\mathrm{d}t^2 + \mathrm{e}^{2m_{11}t}\, \mathrm{d}x_9^2 + \mathrm{d}x^2\right). \tag{14}$$

- *$D = 9$, case 1* with $\{m_{\mathrm{IIA}} = -\frac{2}{3} m_4\}$ has $\Lambda = +\frac{63}{4} {m_4}^2 \, \mathrm{e}^{\phi - 3\varphi/\sqrt{7}}$, which gives rise to de Sitter$_9$, breaking all supersymmetry. This case follows from the reduction of Mink$_{10}$ by using a combination of IIA scale symmetries that leave the dilaton invariant. This particular scale symmetry allows a SS reduction of a configuration with a zero dilaton so that, after reduction, one is left with a non-trivial metric field only.
- *$D = 9$, case 4* with $\{m_{\mathrm{IIB}}, m_3\}$ has $\Lambda = +28 {m_{\mathrm{IIB}}}^2 \, \mathrm{e}^{4\varphi/\sqrt{7}}$, which gives rise to de Sitter$_9$ for non-vanishing m_{IIB}. This case follows from the reduction of Mink$_{10}$ by using a combination of IIB scale symmetries that leave the dilaton invariant. Note that for vanishing m_{IIB} this reduces to Mink$_9$, despite the presence of m_3 [6]. For either m_{IIB} or m_3 non-zero this solution breaks all supersymmetry.

5. Conclusions

We have constructed five different $D = 9$ massive deformations with 32 supersymmetries, each containing two mass parameters. All these five theories have a higher-dimensional origin via SS reduction from $D = 10$ dimensions. Furthermore, the massive deformations gauge a global symmetry of the massless theory. The gauge groups we have obtained are the Abelian groups $SO(2)$, $SO(1,1)^+$, $\mathbb{R}$, $\mathbb{R}^+$ and the unique two-dimensional non-Abelian Lie group $A(1)$ of scalings and translations on the real line.

Not all gauged supergravities we constructed are necessarily the leading terms in a low-energy approximation to (compactified) superstring theory. In particular, the relation to string theory of those massive deformations that are based on a symmetry, that is, broken by α' corrections, is less clear. In contrast, the massive deformations that are based on symmetries that are preserved by the higher-order string corrections to supergravity can be considered as the low-energy limits of compactified string theory. We have two such symmetries:

- The $SL(2,\mathbb{R})$ (or rather its $SL(2,\mathbb{Z})$ subgroup) symmetry of IIB. Thus the $\vec{m} = (m_1, m_2, m_3) \neq 0$ deformations correspond to the low-energy limits of three different sectors of compactified IIB string theory (depending on $\vec{m}^2 = \frac{1}{4}({m_1}^2 + {m_2}^2 - {m_3}^2)$). In [1] vacuum solutions were constructed for all three sectors. Of these only the D7-brane has a well-understood role in IIB string theory.
- The linear combination $m_{\mathrm{IIA}} = \frac{1}{12} m_4 \neq 0$ of SO(1,1)-symmetries of IIA. Thus one can define a massive deformation m_s within case I with $\left\{m_{\mathrm{IIA}} = \frac{1}{12} m_s, m_4 = m_s\right\}$ which

corresponds to the low-energy limit of a sector of compactified IIA string theory. No vacuum solution has been constructed for this sector. It would be very interesting to try to find a vacuum solution and understand what role it plays in IIA string theory.

In fact, one can have a better understanding of the m_s massive deformation of IIA from the following point of view. The particular m_s-deformation is based on a scale symmetry of IIA that can be understood from its 11D origin as the general coordinate transformation $x^{11} \to \lambda x^{11}$. This explains why all α' corrections transform covariantly under this specific scale symmetry: the higher-order corrections in 11D are invariant under general coordinate transformations and upon reduction they must transform covariantly under the reduced g.c.t.'s, among which is the scale symmetry that leads to the m_s-deformation.

The transformation $x^{11} \to \lambda x^{11}$ can also be used for a Scherk–Schwarz reduction from 11D to 9D with a different procedure to give internal coordinate dependence to the fields. Let us call this an SS2 reduction as opposed to the SS1 reduction, which is the method we have used throughout the paper and which is based on *global, internal symmetries* of the higher-dimensional theory. The SS2 procedure [10] instead uses *a symmetry of the compactification manifold* for the reduction ansatz[3]. The massive deformations resulting from a SS2 reduction can be expressed in terms of the structure constants of the corresponding non-Abelian-gauge group. Using the transformation $x^{11} \to \lambda x^{11}$ in the SS2 reduction from 11D to 9D, we obtain massive deformations which are equal to the m_s-deformations upon relating the components of $f_{ab}{}^c$ to m_s. Indeed, this explains why the m_s-deformations correspond to a gauging of the 2D non-Abelian Lie group $A(1)$ rather than only a scale symmetry.

The $D = 9$ gauged supergravities involving m_{11}, m_{IIB} or $m_{\mathrm{IIA}} \neq \frac{1}{12}m_4$ have the same status as the $D = 10$ gauged supergravity discussed above, i.e., these theories are based upon symmetries that are broken by α'-corrections. Note that all the de Sitter space solutions we found in section 4 involve either m_{11}, m_{IIB} or $m_{\mathrm{IIA}} \neq \frac{1}{12}m_4$. It would be interesting to see whether these de Sitter spaces could occur as the $\ell_s \to 0$ limit of an exact solution of string theory.

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